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ON THE THEORY OF FRACTURE OF SOLIDS SUBMITTED TO POWERFUL PULSED ELECTRON BEAMS*

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The irradiation of solids by pulsed (of nanosecond periodicity) relativistic electron beams (also by powerful optic laser beams) led to the discovery of a new type of fracture /1—14/, entirely different from viscous or brittle fracture type produced by mechanical loads /15/. A theory based on the assumption of formation in a solid subjected to such irradiation of clusters of electrons that act as "knives" or "wedges" cutting the solid. Basic model problems of this theory are formulated.

Collective relativistic interactions of faster-than-light electrons of a beam in a medium are considered in Sect.1, where existence of self-compaction of beams of fast moving charged particles is also demonstrated. An exact solution of the steady plane dynamic problem of the elasticity theory of the infinitely thin wedge moving at supersonic velocity (a solution similar to that of the gasdynamic problem of flow over a wedge /16/) is derived in Sect.2. It is then used in Sect.3 for determining the unsteady motion of a wedge of finite length. A simple estimate of the fracture dimension is obtained for high initial wedge velocity.

The first results of investigations on fracturing semiconductor crystals by intensive high energy electron beams were published by Oswald in 1966 /1/. Such electron beams of a density of $10^3\,A/m^2$ are generated in electron guns at voltages up to 10 MV with pulses at $10^{-9}+10^{-7}$ s and frequencies of hundreds of Hertz. Subsequent works disclosed and investigated the fracturing of such diverse materials as metals, dielectrics, ion crystals, glass, and various rocks.

Analysis of experimental results made it possible to establish the following particularities of the fracture process: a) fracturing of all materials (including the highly plastic under mechanical loads) is of the "brittle" type, i.e. the specimen appears to have been split by a crack without any trace of permanent set; b) initial microdefects and cracks (even fairly large) do not affect the fracture threshold, the beam intensity (the density of absorbed energy) at which splitting of the specimen takes place; c) the fracture threshold — a constant of the material—is the minimum irradiation intensity capable of inducing fracture; d) the crack that fractures the specimen propagates at supersonic velocity, and e) the fracture threshold is independent of temperature and purity of crystals, as well as of the energy of electrons in the beam (in the range of 0.5— 10 MV). These effects are entirely extraneous to the usual viscous, brittle, or mixed mechanical fracture. So far there is no theoretical explanation of this phenomenon.

The problem of splitting a brittle body by a thin wedge of arbitrary form was considered in /17/ whose most important conclusion was that the propagation velocity of cracks ahead of the wedge cannot exceed the Rayleigh wave velocity (always lower than the velocity of longitudinal waves). Later, the motion of a thin wedge at a velocity higher than the Rayleigh one but lower than that of transverse waves was investigated in /18/. It was found that the wedge was in contact with the body along some finite segment of its frontal part, while its remaining surface was free. Supersonic crack /propagation/ under gigantic pulses of irradiation by electron and laser beams, evidently presupposes the existence of some macroscopic objects which cut the body at supersonic velocity. It is reasonable to assume that in both cases (in what follows only high-energy pulsed electron beams are considered) there is some physical mechanism that induces the formation of clusters of solid body electron or electron-proton plasma, which act as "knife edges" cutting the body. (According to certain estimates /12,19/ the mean-square propagation velocity of solid-body plasma fluctuations is of the order of 10^6 m/s.) In the case of electron irradiation one of such mechanisms can be the self-compaction of faster-than-light electrons, which comes into existence after the initial beam density has reached a certain value.

This hypothesis is accepted below, since it makes possible the explanation and understanding the indicated distinctive features of fracture by electron beams. The laws of motion of the fracturing plasma wedge in the solid are, then, determined on very rough assumptions that the wedge is absolutely rigid and the body perfectly elastic (irreversible deformations at the crack edges cannot develop at high velocities), homogeneous, and isotropic. The resulting

*Prikl.Matem.Mekhan.,44,No.6,1120-1128,1980 mathematical problem is similar to that of the flow of a supersonic gas over a thin wedge, except that it is somewhat more complicated, as will become subsequently clear, owing to the presence of two wave equations in the system.

1. Collective relativistic interactions in electron beams.

The proper electromagnetic field of an electron with a charge $e < 0$ moving in a medium along axis z at constant velocity V higher than the phase speed of light a, but lower than the velocity of light c in vacuum /20/, in its proper reference system (with allowance for Lorentz transformations) is of the form

$$E_z = \frac{-eM^2 z}{2\pi\varepsilon' (z^2 - M^2 r^2)^{3/2}}, \quad E_i = \frac{eM^2 x_i}{2\pi\varepsilon' (z^2 - M^2 r^2)^{3/2}}, \quad (r^2 = x_1^2 + x_2^2;\ i = 1, 2) \tag{1.1}$$

$$B_z = 0, \quad B_i = \frac{\mu' e V M^2 (1 - a^2/c^2)\, x_{i-1}}{2\pi (1 - V^2/c^2)(z^2 - M^2 r^2)^{3/2}}, \quad (x_{i-1} = x_2, -x_1) \tag{1.2}$$

$$M^2 = \frac{V^2/a^2 - 1}{1 - V^2/c^2} = \frac{\mu\varepsilon V^2/c^2 - 1}{1 - V^2/c^2} > 0, \quad a = \frac{c}{\sqrt{\mu\varepsilon}}, \quad \mu' = \mu\mu_0, \quad \varepsilon' = \varepsilon\varepsilon_0$$

where $E_1, E_2, E_z, B_1, B_2, B_z$ are components of the electromagnetic field (SI) in the system of coordinates $x_1 x_2 z$ attached to the electron, μ' and ε' are, respectively, the permeability and permittivity of the medium, and M is the relativistic Mach number. Field (1.1), (1.2) lies in the Mach cone of the faster-than-light electron $z^2 > M^2 r^2, z < 0$ outside which the electron has no proper field.

Dissipation of the electron energy (and its retardation) is the result of its field interaction with the medium electromagnetic field (braking radiation losses and excitation of bound electrons of the substance), wave losses at the Mach cone front (Cherenkov radiation), and of the interaction of its field with the electromagnetic field of other electrons of the beam (collective interaction). At high velocities the first two kinds of losses are the same for all electrons of the beam and do not affect the relative position of electrons. Since the relative position of electrons in the beam is, thus, determined by collective interactions, only they are considered below.

Using the method of invariant Γ-integrals (as was done in the case of the slower-than-light electron in the external field /21/), and (1.1) and (1.2) it is possible, in the case of the faster-than-light electron in the external field $\mathbf{E}_0 = \{E_{0i}\}, \mathbf{B}_0 = \{B_{0j}\}$, to obtain

$$\Gamma_i = eE_{0i} \quad (i = 1, 2, z) \tag{1.3}$$

where Γ_i is the irreversible work of the external field for a unit length translation of the electron along the i-th axis. When in the external field $\mathbf{B}_0 = 0$, Γ_i are components of the force acting on a charge. This cannot be taken as self-evident in the case of faster-than-light velocities; the method of Γ-integrals which represent a form of notation of the general laws of conservation, enables us to provide a strict substantiation of the above formula.

Note that the external field $\mathbf{E}_0, \mathbf{B}_0$ is considered in its proper coordinate system attached to the moving electron.

Let in the trail (the Mach cone) of the leading electron e_0 moving in the medium at a faster-than-light velocity $V > a$, be another electron e_1 for which the external field is the field (1.1), (1.2) of electron e_0. Electron e_1 is obviously always attracted to the leading electron, i.e. $\Gamma_z = e_1 E_{0z} > 0$. When two electrons move along the z-axis, the attraction is

$$F_1(z) = \Gamma_z = \frac{e^2 M^2}{2\pi\varepsilon' z^2} \tag{1.4}$$

where z is the distance between the electrons.

Note that, since electron e_1 interacts with the lagging field of electron e_0 the latter is unaffected by electron e_1.

A simple estimate of the behavior of a relativistic system of electrons can be obtained by confining the analysis to the model of one-dimensional semi-infinite chain of electrons which at the initial instant of time are equally spaced at intervals b. A simple analytic solution can be obtained for the determination of motion of the single electron e_1 in the field of electron e_0. In the one-dimensional system forces can act only along the chain axis. We denote by f_{mn} the force exerted by the n-th electron on the m-th electron ($n < m$). The resultant of all forces acting on the m-th electron is

$$F_m = \sum_{n=0}^{m-1} f_{mn} \tag{1.5}$$

In accordance with (1.1), (1.3), and (1.5) we have at the initial instant of time

$$F_m(b) = \frac{e^2 M^2}{2\pi\varepsilon' b^2} \sum_{n=0}^{m-1} (n+1)^{-2}$$

Using the estimate of /22/ for the sum in the right-hand side of this equality we obtain

$$F_1(b) \leqslant F_m(b) < \pi^2 F_1(b) / 6$$

which shows that $F_m(b)$ differs only little from $F_1(b)$ for any m.

The relativistic differential equation of electron motion is according to (1.4) in the attached reference system of the form /23/

$$\frac{d^2z}{dt^2} = \frac{e^2(V^2/a^2-1)}{2\pi\varepsilon' m_0 z^2(1-V^2/c^2)}$$

Its solution with the following initial conditions for $t=0$: $z=-b$, $dz/dt=0$ yields

$$tK^{1/2} = [-bz(z+b)]^{1/2} + b^{3/2}\arcsin[(z+b)/b]^{1/2}, \quad K = \frac{e^2(V^2/a^2-1)}{\pi\varepsilon' m_0(1-V^2/c^2)} \tag{1.6}$$

Let us estimate the characteristic time τ in which electron e_1 converges with e_0 (a compact system of two electrons is formed in which quantum interactions that are not accounted for in the solid medium model, play the determining part, and the retarding forces due to radiation of the accelerated electron; the applicability limit of solution (1.6) for the beams used (in practice) may be roughly estimated using distances of order 10^{-13} m). Setting $z=0$ we obtain (1.7).

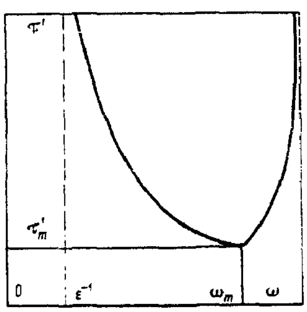

Fig.1

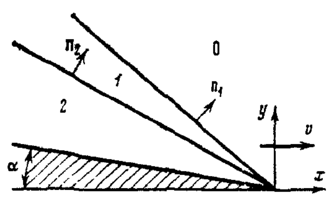

Fig.2

$$\tau = \frac{\pi b^{3/2}}{2K^{1/2}} \tag{1.7}$$

Note that the quantities b, t and τ are considered in the moving electron proper coordinate system. Passing to the laboratory /coordinate/ system using the Lorentz transform $b' = b(1-V^2/c^2)^{1/2}$, $t' = t(1-V^2/c^2)^{-1/2}$, from (1.7) we obtain

$$(\tau')^2 = \frac{\pi^3\varepsilon_0 m_0 (b')^{3/2}}{4e^2\mu(1-V^2/c^2)^{3/2}[V^2/c^2-(\mu\varepsilon)^{-1}]} \tag{1.8}$$

The dependence of τ' on $\omega = V^2/c^2$ for some constant $\mu\varepsilon$ is shown in Fig.1. It will be seen that the effect of convergence is the most pronouned in that range of particle velocities (energies) where the characteristic time τ' is less than the time of existence of the directed faster-than-light beam in the medium. When $\omega_m = (2\mu\varepsilon+3)/(5\mu\varepsilon)$, the time τ' reaches its minimum value

$$\tau_m' = \frac{C_m(b')^{3/2}}{\mu^{1/2}(1-\mu\varepsilon^{-1})^{5/4}}, \quad C_m = \left[\frac{5^5\pi^6 m_0^2\varepsilon_0^2}{12^3 e^4}\right]^{1/4} \tag{1.9}$$

where for electron beams $C_m = 6{,}437\cdot 10^{-2}$ m$^{-3/2}$ s. For example, when $b' \sim 10^{-6}$ m (this is the order of distances in the applied pulsed electron beams), we obtain $\tau_m' \sim 10^{-10}$ s.

As shown on the simple model, a mechanism of self-compaction exists in relativistic electron beams in a medium (theoretically this effect holds for any beams of uniformly charged particles of corresponding energies). Since time T is assumed small in the problem formulation, the effect of that mechanism can make itself felt only in the case of beams of fairly high initial intensity, i.e. small b'. The initial beam density needed for the formation of a dense bunch in a dielectric medium $(\mu=1)$ can in accordance with formula (1.9) be estimated as

$$g \sim (b')^{-3} > C_m^2(1-\varepsilon^{-1})^{-5/2}T^{-2} \tag{1.10}$$

The time T of existence of a beam of faster-than-light electrons in a medium is determined by two factors, viz. deceleration of electrons to the velocity of light and by the losses due to excitation and ionization of the substance bound electrons /24/. A more exact estimate of critical density in the case of high energies $(\omega \gg \omega_m)$ can be obtained from formula (1.8).

It should be pointed out that the initial density calculated by formula (1.10) is higher than the usually attained mean density. However, since time T is considerably shorter than the accelerator pulse duration, dimensions of the critical density region may be considerably smaller than dimensions of the beam. In small volumes large densities can be due to nonuniform density distribution (usually considerable /11,14/) in the radial and axial directions, as well as to the stream of ionized electrons /24/ and to the possibility of large fluctuations in small volumes.

2. **Steady supersonic motion of an infinite wedge.** Let an infinitely thin wedge with an apex angle of 2α move in an elastic medium at constant supersonic velocity v (Fig.2). In the system of coordinates xy attached to the wedge the equations of the steady plane problem of the dynamic theory of elasticity are of the form

$$M_i^2 \frac{\partial^2\Phi_i}{\partial x^2} = \frac{\partial^2\Phi_i}{\partial y^2} \quad \left(i=1,2;\ M_i^2 = \frac{v^2}{c_i^2} - 1 > 0\right), \quad u_x = \frac{\partial\Phi_1}{\partial x} + \frac{\partial\Phi_2}{\partial y} - vt, \quad u_y = \frac{\partial\Phi_1}{\partial y} - \frac{\partial\Phi_2}{\partial x} \tag{2.1}$$

$$\frac{\sigma_{xx}}{\mu} = (1 + M_2^2 - 2M_1^2)\frac{\partial^2\Phi_1}{\partial x^2} + 2\frac{\partial^2\Phi_2}{\partial x\partial y}, \quad \frac{\sigma_{yy}}{\mu} = (M_2^2 - 1)\frac{\partial^2\Phi_1}{\partial x^2} - 2\frac{\partial^2\Phi_2}{\partial x\partial y} \tag{2.2}$$

$$\frac{\sigma_{xy}}{\mu} = 2\frac{\partial^2\Phi_1}{\partial x\partial y} + (M_2^2 - 1)\frac{\partial^2\Phi_2}{\partial x^2}$$

where Φ_1 and Φ_2 are wave potentials, u_x and u_y are displacements, σ_{xx}, σ_{yy}, and σ_{xy} are stresses, M_i are modified Mach numbers, μ is the Lamé constant of the elastic medium, and c_1 and c_2 are the velocities of longitudinal and transverse elastic waves.

In terms of wave potentials the velocities of the medium are of the form

$$v_x = -v\left(1 + \frac{\partial^2\Phi_1}{\partial x^2} + \frac{\partial^2\Phi_2}{\partial x\partial y}\right), \quad v_y = -v\left(\frac{\partial^2\Phi_1}{\partial x\partial y} - \frac{\partial^2\Phi_2}{\partial x^2}\right) \tag{2.3}$$

The general solution of Eq.(2.1) is

$$\Phi_i = \varphi_i(x - M_i y) + \psi_i(x + M_i y) \tag{2.4}$$

where φ_i and ψ_i are arbitrary twice differentiable functions. Potentials Φ_i are determined in regions bounded by sets of Mach lines $x \pm M_i y = 0$ and the wedge surface. We denote parameters of the unperturbed medium (where the displacements and stresses are zero), by subscript 0, those of the medium between the Mach lines $x + M_1 y = 0$ and $x + M_2 y = 0$ by subscript 1 and those between $x + M_2 y = 0$ and the wedge surface by subscript 2 (Fig.2).

The conditions of conservation of mass and momentum at discontinuities — the Mach lines —are (owing to symmetry) we restrict the analysis to the upper half-plane, of the form along $x + M_1 y = 0$

$$\rho_0 v_{01} = \rho_1 v_{11} \tag{2.5}$$
$$\sigma_{01} - \sigma_{11} = \rho_0 v_{01}(v_{01} - v_{11}), \quad \tau_{01} - \tau_{11} = \rho_0 v_{01}(w_{01} - w_{11})$$

and along $x + M_2 y = 0$

$$\rho_1 v_{12} = \rho_2 v_{22} \tag{2.6}$$
$$\sigma_{12} - \sigma_{22} = \rho_1 v_{12}(v_{12} - v_{22}), \quad \tau_{12} - \tau_{22} = \rho_1 v_{12}(w_{12} - w_{22})$$

where ρ is the medium density, v_{lk} and w_{lk} are the components of the medium velocity normal and tangent to Mach lines (with the normal $\mathbf{n}_k$) in region l, and σ_{lk} and τ_{lk} are the normal and tangent components of the stress vector in area elements with the normal $\mathbf{n}_k$ in region l.

We express, in conformity with (2.2)—(2.4), the quantities v_{lk}, w_{lk}, σ_{lk} and τ_{lk} in terms of second derivatives φ_i'' and ψ_i'' of potentials. From conditions (2.5) at the discontinuity of $x + M_1 y = 0$ we obtain

$$\varphi_1'' = 0, \quad \rho_1 = \rho_0[1 + (1 + M_1^2)\psi_1''(0)]^{-1} \tag{2.7}$$

(since $\mu = \rho_0 c_2^2$, the second condition (2.5) is an identity).

From conditions (2.6) on $x + M_2 y = 0$ with allowance for (2.7) we obtain

$$\varphi_2'' = 0, \quad \rho_2 = \rho_1 \tag{2.8}$$

Smallness of the wedge apex angle has been taken here into account, and the density of the medium in each of the regions 0, 1, 2 was assumed constant.

It is evident that under conditions (2.7) and (2.8) the displacements are everywhere continuous.

The substitution of (2.7) and (2.8) into (2.2) and (2.3) yields

$$v_x = -v\,[1 + \psi_1'' + M_2\psi_2''], \quad v_y = -v\,[M_1\psi_1'' - \psi_2''] \tag{2.9}$$
$$\sigma_{xx} = \mu\,[(1 + M_2^2 - 2M_1^2)\psi_1'' + 2M_2\psi_2'']$$
$$\sigma_{yy} = \mu\,[(M_2^2 - 1)\psi_1'' - 2M_2\psi_2''], \quad \sigma_{xy} = \mu\,[2M_1\psi_1'' + (M_2^2 - 1)\psi_2'']$$

We recall that in formulas (2.9) $x + M_i y$ serve as arguments of functions ψ_i'', and that in regions 0 it is necessary to set $\psi_1'' = 0$ and $\psi_2'' = 0$, but $\psi_2'' = 0$ in region 1.

At the wedge surface $y + \delta x = 0$ $(\delta = \mathrm{tg}\,\alpha)$ the velocities and stresses are of the form

$$v_{2\delta} = -v\,(1 + \delta^2)^{-1/2}\,[\delta + (M_1 + \delta)\psi_1'' + (M_2\delta - 1)\psi_2''] \tag{2.10}$$
$$w_{2\delta} = -v\,(1 + \delta^2)^{-1/2}[(M_1\delta - 1)\psi_1'' - (M_2 + \delta)\psi_2'' - 1]$$

$$\sigma_{2\delta}/\mu = \{[(1 + M_2^2 - 2M_1^2)\delta^2 + 4M_1\delta + M_2^2 - 1]\psi_1'' + 2\,(M_2\delta - 1)(M_2 + \delta)\psi_2''\}(1 + \delta^2)^{-1}$$

$$\tau_{2\delta}/\mu = \{2\,(M_1 + \delta)(M_1\delta - 1)\psi_1'' + [(M_2^2 - 1)\delta^2 - 4M_2\delta + 1 - M_2^2]\psi_2''\}(1 + \delta^2)^{-1}$$

where $(M_i - 1/\delta)y$ is the argument of functions ψ_i''.

The absence of normal velocity components of the medium on the wedge surface implies in conformity with (2.10) that

$$(1 - M_2\delta)\psi_2''\,(M_2 y - y/\delta) - (M_1 + \delta)\psi_1''\,(M_1 y - y/\delta) = \delta \tag{2.11}$$

The second boundary condition at the wedge surface is generally of the form

$$\tau_{2\delta} = F\,(w_{2\delta},\ \sigma_{2\delta}) \tag{2.12}$$

where F is a function obtained from experimental data or model concepts. It is, for instance, possible to assume that $\tau_{2\delta} = f\sigma_{2\delta}$, where f is the friction coefficient.

Conditions (2.11) and (2.12) enable us to determine functions ψ_1'' and ψ_2'', and completely solve the stated here problem of determination of the velocity and stress fields (2.9) at supersonic motion of the wedge in an elastic medium. It will be seen that in each of regions 1 and 2 the velocity and stress field is piece-wise constant (as in the similar gasdynamical problem of flow over a wedge /16/). We present the final formulas only for the simplest and important case, restricting to the minimum the use of data on insufficiently known properties of supersonic clusters.

Ignoring friction $(F = 0)$ and restricting the analysis to very small apex angles of the wedge $(\delta \ll 1/M_2)$, from (2.9)—(2.12) we obtain

$$\psi_1'' = \frac{(1 - M_2^2)\,\delta}{(1 + M_2^2)(M_1 + \delta)}, \quad \psi_2'' = \frac{2\delta}{1 + M_2^2}, \quad \rho_1 = \rho_2 = \rho_0\left[1 + \frac{(M_2^2 - 1)\,\delta}{M_1 + (2 - M_2^2)\,\delta}\right] \tag{2.13}$$

The velocity and stress fields can be determined by substituting (2.10) into (2.13), which yields

$$\sigma_{2\delta} = -\frac{\mu\delta\,[(M_2^2 - 1)^2 + 4M_1M_2 + 4M_2\delta]}{(M_1 + \delta)(1 + M_2^2)}, \quad \tau_{2\delta} = 0 \tag{2.14}$$

3. The braking of a finite edge in quasisteady approximation.

Let a thin wedge-shaped body of characteristic length L and maximum thickness $h = 2L\,\mathrm{tg}\,\alpha = 2L\delta)$ move in an elastic medium. The lateral surface of the wedge interacts along that length with the elastic medium, while the fracture cavity surface beyond that length is free (Fig.3). At the initial instant of time $t = 0$ the wedge-shaped body momentum was $P_0 = mv_0$, where v_0 is the initial velocity, and m and P_0 are, respectively, the mass and momentum per unit of the wedge width.

Let us consider the supersonic motion of the wedge-shaped body subjected to the resistance of an elastic medium, disregarding friction forces and the action of the substance in the cavity. The resistance acting on the wedge (per unit of width) is

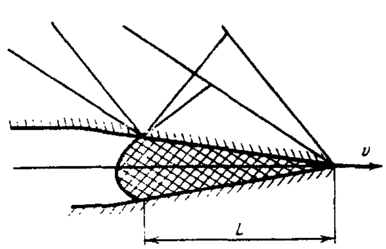

Fig.3

$$R = 2\delta L\sigma_{2\delta}$$

which in the quasisteady approximation (2.14) yields

$$R = -\frac{2\mu\delta^2 L\,[(M_2^2-1)^2+4M_1M_2+4M_2\delta]}{(M_1+\delta)(1+M_2^2)}$$

The solution of the equation of wedge motion $m\,dv/dt = R$ with initial condition $v = v_0$ at $t = 0$ is of the form

$$t = m\int_{v_0}^{v} R^{-1}dv$$

For the solution in the most important limit case of $v^2/c_1^2 \gg 1$ we have

$$t = \frac{m}{2\rho_0\delta^2 L c_1}\ln\frac{v_0}{v}$$

The corresponding distance travelled during time t by the wedge is

$$D(t) = \int_0^t v\,dt = \frac{m v_0}{2\rho_0\delta^2 L c_1}\left[1-\exp\left(\frac{-2\rho_0\delta^2 L c_1 t}{m}\right)\right]$$

Since $m \simeq \rho_w\delta L^2$, where ρ_w is the density of the wedge material (electron plasma), hence

$$\frac{D_\infty}{L} = \frac{\rho_w v_0}{4\delta\rho_0 c_1} \tag{3.1}$$

Formula (3.1) provides a simple estimate of dimensions of the crack generated by the supersonic motion of the wedge, and is a unifying characteristic of the mechanical model of supersonic fracture.

Remark on three-dimensional problems. Clusters of electron plasma are of finite dimensions in all directions. Hence three-dimensional problems of the supersonic motion of slender wedges of particular cross-sections (such as triangular, circular, etc.) in an elastic body, are of interest. The most important of these are the problems of supersonic mathematical branch cuts, in connection with which arises the problem of determination of crack detachment from the wedge profile, typical of hydrodynamic detached flows.

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